MAGNETIC FIELD AND MAGNETIC FORCES

We saw in previous chapter that electric force arise in two stages:

(1) a charge produces an electric field in the space around it

(2) a second charge responds to the electric field.

The fundamental origin of magnetism is the interaction of moving electric charges. Unlike electric forces which act on electric charges whether they are moving or not, magnetic forces act only on moving charges.

Magnetic forces arise in two stages:

(1) a moving charge, or a collection of moving charges (electric current) produce a magnetic field.

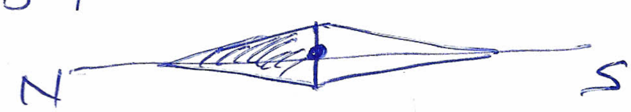
(2) a second current or moving charge respond to this magnetic field and experiences a force.

① Magnetism

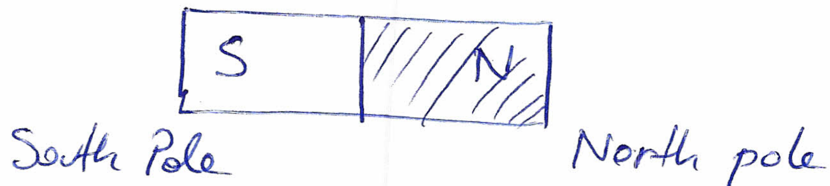
Magnetic phenomena first observed about 2500 years ago in fragments of magnetized iron (Fe) near the ancient city of Magnesia (Turkey). These fragments are examples of what we call PERMANENT MAGNETS.

Permanent magnets exert forces on each-other as well as on pieces of Fe non-magnetized initially but which become magnetized in proximity of a permanent magnet.

Another example of permanent magnet is the compass needle which tends to line itself along the geographic N-S direction



Before the relationship of magnetic interaction of moving charges was understood, the interaction of magnets were described in terms of magnetic poles.



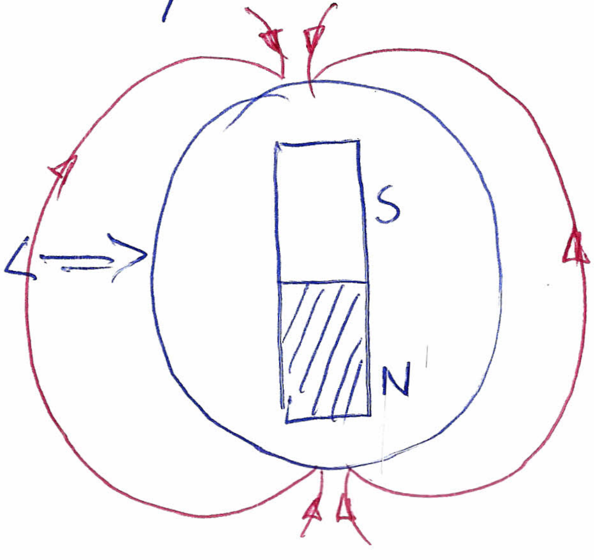
Magnetic interactions

opposite pole attracts | like-pole repel

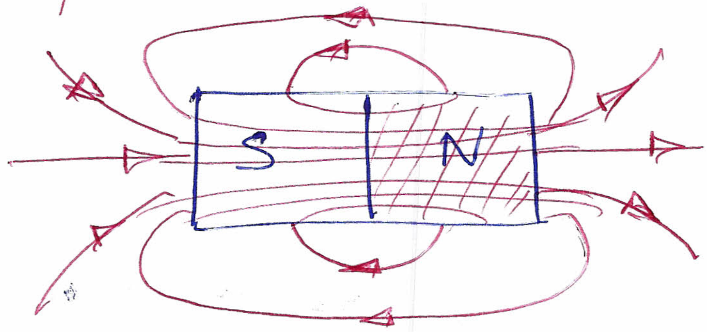
An object that contains Fe (initially not magnetized) is attracted by any pole of a magnet

The nail get magnetized: the magnet creates a magnetic field and the nail responds to that field

The Earth itself it's a big magnet. Its North geographic pole is close to a SOUTH magnetic pole. This is why the compass north pole indicates the geographic North.



magnetic field lines



get out from N and enter in S pole.

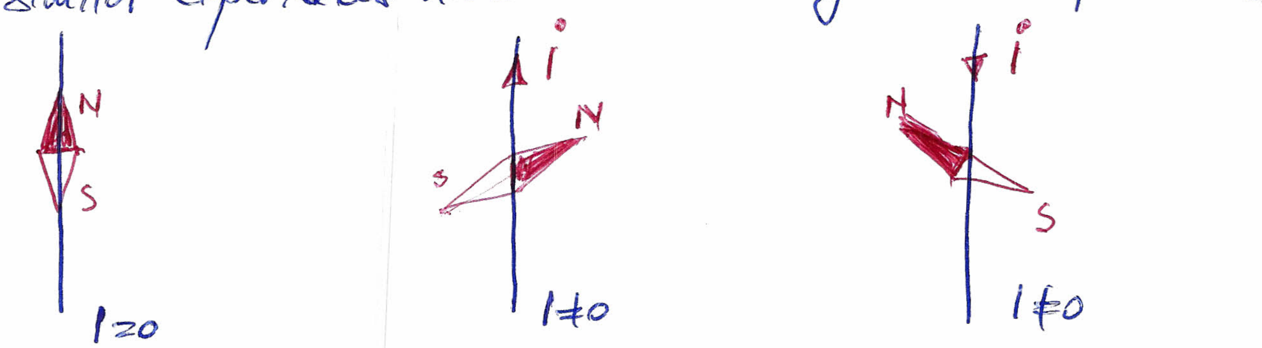
magnetic poles versus electric charge

→ Isolated positive and negative charges exist

→ no experimental evidence of single isolated magnetic poles. ⇒ POLES ALWAYS APPEAR IN PAIR. When we broke in two a magnet bar, each end becomes pole

⇒ magnetic monopoles does not exist... despite extensive studies for searching them.

First evidence of relationship between magnetism and moving charges was discovered in 1820 by the Danish scientist Hans Christian Oersted. He found that a compass needle was deflected by a current carrying wire. Similar experiences have been done by André Ampère (FR)



Hystorically, Michael Faraday (UK), Joseph Henry (USA) descovered that moving a magnet near a conducting loop can cause a current through the loop.

We know now that the magnetic forces between two bodies are due to interactions between moving electrons in the atoms of bodies. (There are also electric interactions between bodies but these are much weaker because the two bodies are electrically neutral).

Inside a magnetized body (permanent magnet) there is a coordinated motion of some electrons in atoms; in unmagnetized bodies these motions are not coordinated. — we will see in detail later —

Electric and magnetic interactions are intimately connected. The unified principles of magnetism are expressed by the Maxwell's equations as we will see later.

[2] Magnetic field

Similarly to the formulation of electric interaction:

(1) a distribution of electric charges at rest creates an electric field $\vec{E}$ in the surrounding space

(2) this electric field $\vec{E}$ exerts a force $\vec{F} = q\vec{E}$ on any charge q present in the field

we describe the magnetic interactions as

(1) A moving charge or a current creates a MAGNETIC FIELD in the surrounding space (in addition to its electric field.

(2) The magnetic field exerts a force $\vec{F}$ on any moving charge or current that is present in the field.

In this chapter we concentrate on the second aspect of the interaction: given a magnetic field, what force does it exert on a moving charge or current?

Magnetic force on moving charges (Lorentz force)

The magnetic force $\vec{F}$ acting on a positive charge q moving with velocity $\vec{v}$ in a magnetic field is

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

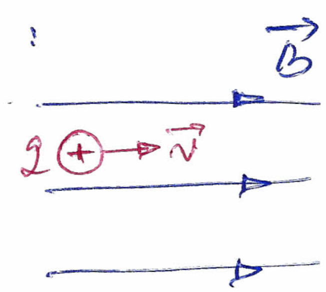
Lorentz force

- the magnitude: $F = |q| v B \sin\alpha$

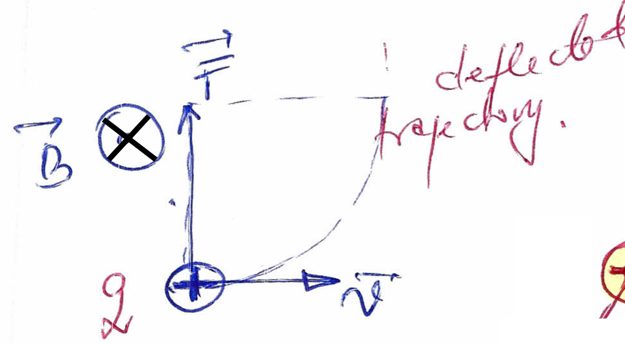
$$= |q| v_{\perp} B$$

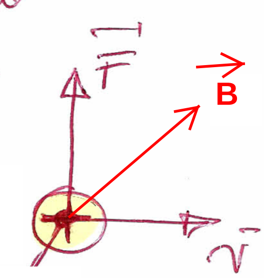
- the direction: right-hand rule

ex:



$F = 0$ $(\vec{v} \parallel \vec{B})$





for $\vec{v} \perp \vec{B}$ a net $\vec{F}$ appears which deflects the electron's trajectory

$$[B]_{SI} = \frac{[F]_{SI}}{[q]_{SI}[v]_{SI}} = \frac{N\,s}{C\,m} = \frac{N}{A\,m} = 1\,T \text{ (TESLA)}$$

($\frac{C}{s} = A$, i.e. $\frac{s}{C} = \frac{1}{A}$)

(Nikola Tesla 1856–1943)

Another unit:

$$1 \text{ Gauss} = 10^{-4} \text{ Tesla}$$

ex: Earth: $B \simeq 10^{-4} T \simeq 1$ Gauss

$B = 45 T$ → larger magnetic field ever produced in laboratories.

120 T short pulses (ms) can be produced.

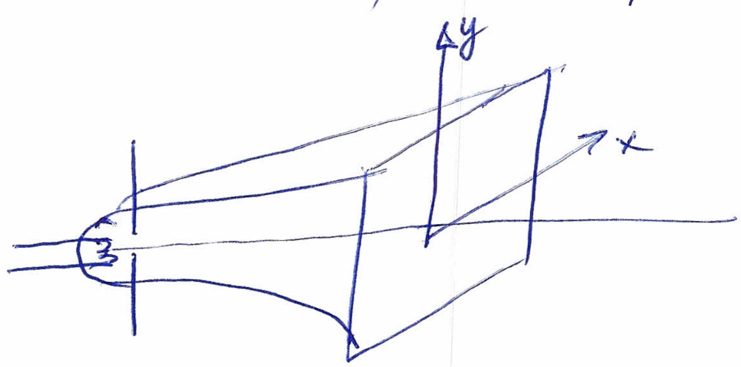
When a charged particle moves through a space region where both $\vec{E}$ and $\vec{B}$ are present, both exerts forces ⇒

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$\vec{E}$ → accelerates

$\vec{v} \times \vec{B}$ → deflects trajectory

ex: cathodic tube, oscilloscope



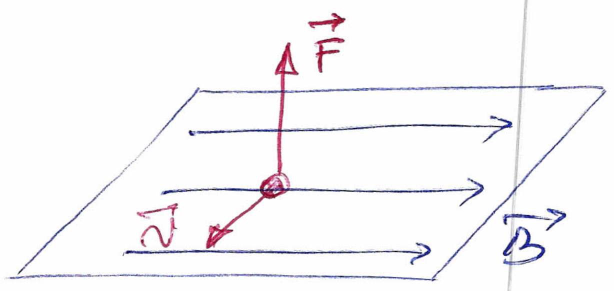
③ Magnetic field lines and magnetic flux

→ as we did for electric field, we draw lines that are, in any point, tangent to $\vec{B}$ in that point.

→ when lines are closed together $\vec{B}$ is large
far apart $\vec{B}$ is small

→ field lines never intersect.

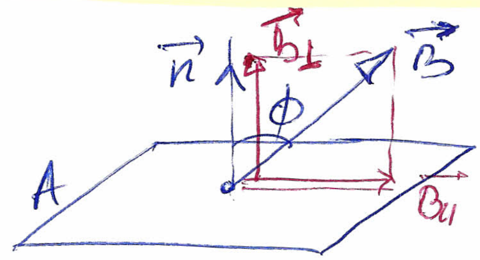
Obs: Unlike electric field lines, magnetic field lines are not lines of force: The force on a moving charged particle is always ⊥ to $\vec{B}$.



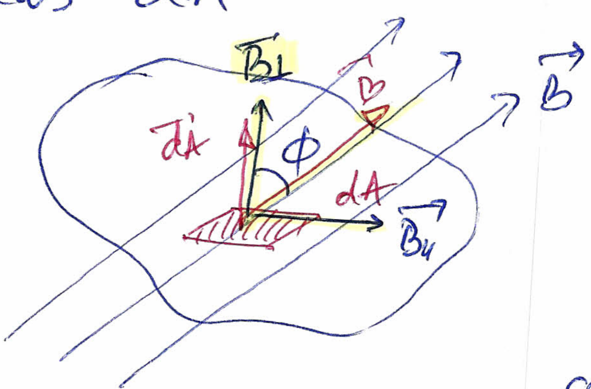
$$\vec{F} = q\vec{v} \times \vec{B}$$

Magnetic flux and Gauss' law for magnetism

$$\Phi_B = B_\perp A = \vec{B} \cdot \vec{A}$$



When we divide a complex surface in elementary areas dA



$$d\Phi_B = B_\perp dA = B\cos\phi \, dA = \vec{B} \cdot d\vec{A}$$

$$\Phi_B = \oint\!\!\oint B_\perp dA = \oint\!\!\oint \vec{B} \cdot d\vec{A}$$ total magnetic flux through a surface

$$[\Phi_B]_{SI} = 1\,T \cdot m^2.$$

In Gauss's law for electric field:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Q_{encl} = total electric charge enclosed by the surface

if closed surface encloses a dipole
⇒ total flux is zero because $Q_{encl} = q_+ + q_- = 0$

By analogy, because we cannot have magnetic monopoles, ⇒

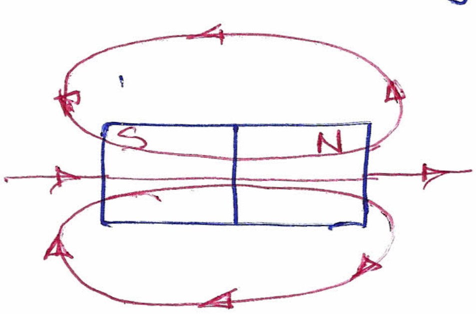
$$\oint \vec{B} \cdot d\vec{A} = 0$$

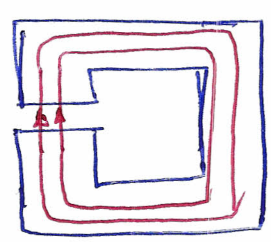
The total magnetic flux to any closed surface is zero.

Gauss's law for magnetism

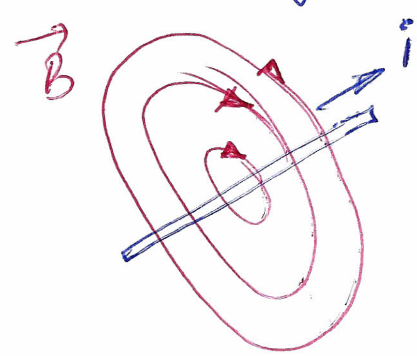
Obs: Magnetic field lines have no ends unlike electric fields that start on positive charges and end on negative charges. Because no magnetic monopoles.

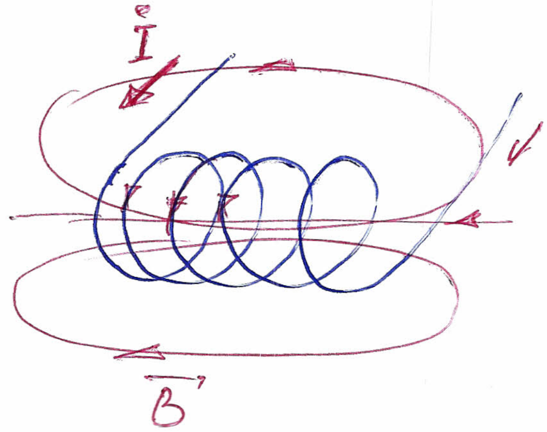
⇒ when representing magnetic field lines they enter in magnets, too.





C-shaped magnet





solenoid ⇔ bar magnet

Obs: Because from $d\phi = B\,dA \Rightarrow$
$$B = \frac{d\phi}{dA} = \frac{\text{flux}}{\text{unit area}}$$
sometimes B is called MAGNETIC FLUX DENSITY

(h) Motion of charged particles in magnetic field

$$\vec{F} = q\vec{v} \times \vec{B}$$
the motion is governed by Newton's laws.

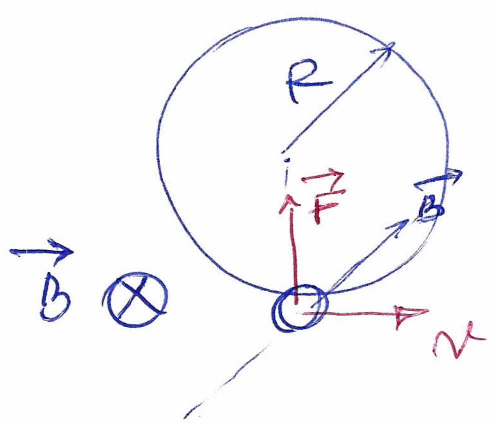
The charged particle moves in the plane perpendicular to $\vec{B}$ with constant speed $|\vec{v}|$

$\vec{F}$ is a centripetal force causing circular motion (when $\vec{v} \perp \vec{B}$)

$$F = |q| v B = m\frac{v^2}{R} \Rightarrow$$

$$\boxed{R = \frac{mv}{|q|B}}$$

clockwise movement of positive charge



Obs (Because $\vec{F} \perp \vec{v}$ the $\vec{B}$ does not work on the particle $\Rightarrow$ $|\vec{v}| = ct$)

$$\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}$$

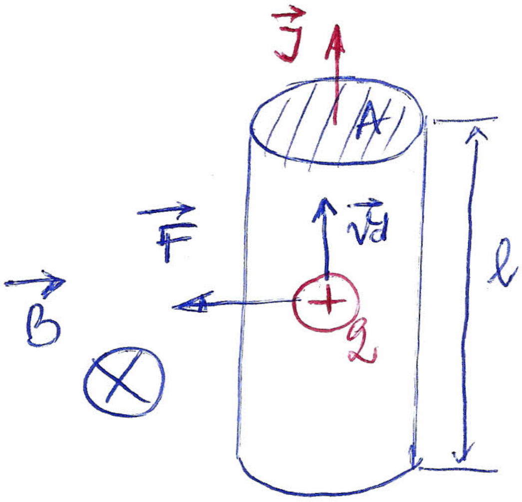
$$\boxed{f = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B}}$$

cyclotron frequency

Applications

- seminary:
  - discuss synchrotron accelerator
  - Van Allen radiation belts (auroras) Earth protection
  - mass spectrometers / velocity selector.

5 Magnetic force on a current-carrying conductor.



on each moving charge q we have the force

$$\vec{F} = q\vec{v}_d \times \vec{B}$$

$\vec{v}_d$ = drift velocity

if $v_d \perp B \Rightarrow F = q v_d B$

The total force for the total moving charges within the volume lA (n = charge density)

$$F = (nAl)\, q v_d B = (n q v_d A)\, l B = I l B$$

$$\Rightarrow \boxed{F = BIl}$$

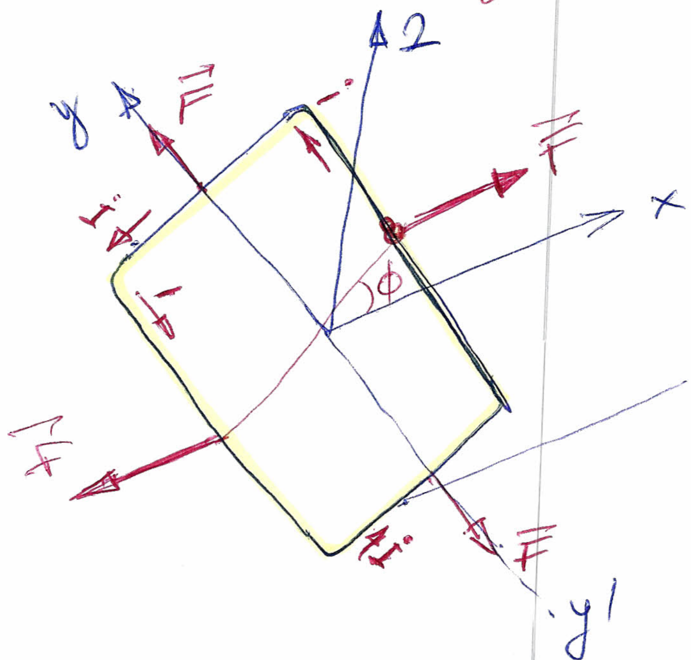
If $\vec{B}$ is not $\perp$ to the wire but makes an angle ϕ

$$\Rightarrow \boxed{\vec{F} = I\vec{l} \times \vec{B}}$$

If the conductor is not straight, we can divide it into infinitesimal segments:

$$\boxed{d\vec{F} = I\, d\vec{l} \times \vec{B}}$$

6. Force and torque on a current loop

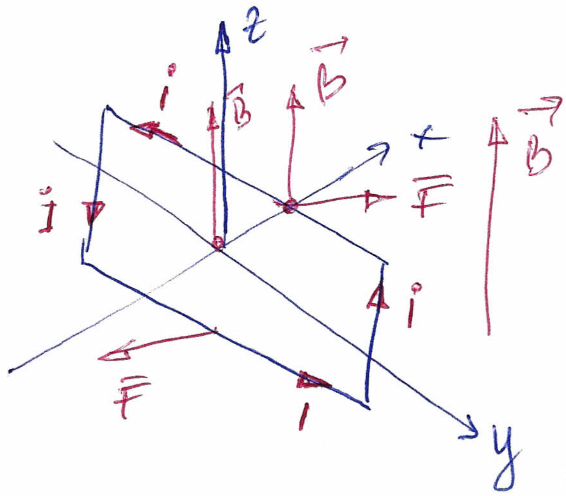


The two forces pairs acting on loop cancels. However, the total torque is non-zero

(e.g. with respect to the rotation axis yy')

if $\phi = 90°$ ($\vec{B}$ is in plane of the loop) the torque is maximum.

$\phi = 0$ B ⊥ to the plane of the loop ⇒ zero torque



Max. torque $\phi = 0$

total force = 0

total torque ≠ 0

$$\boxed{\tau = IBA \sin\phi}$$

ϕ angle between $\vec{n}$ (normal on the loop) and $\vec{B}$

The product $\boxed{IA = \mu}$ magnetic dipole moment

Vector form

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

vector torque on a current loop

Potential energy for a magnetic dipole

When a magnetic dipole changes orientation in the $\vec{B}$ field, the field does work on it. In an infinitesimal angular displacement: $d\phi \Rightarrow$

$dW = \tau d\phi$ and there is a corresponding change in potential energy

By analogy with electric field:

on electric dipole: $\vec{p}$ the torque is $\vec{\tau} = \vec{p} \times \vec{E}$ and dipole potential energy $U = -\vec{p} \times \vec{E}$

$\Rightarrow$

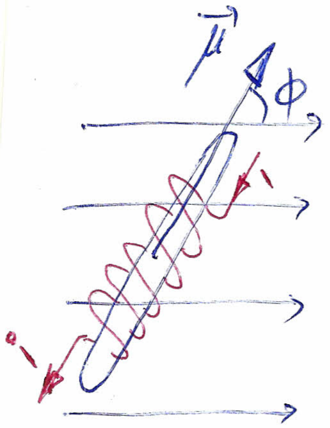
on magnetic dipole: $\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}} \Rightarrow$

$$\boxed{U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi}$$

Obs: All the above equation deduced for rectangular current loop are valid for any plane-shaped loop

Generalization for a solenoid

= N planar loops together



$\Rightarrow \tau = NIAB \sin\phi = \mu B \sin\phi$

$\mu = NIA$

Application NMR (Nuclear magnetic resonance)

The nucleus of each H atom in tissues has a magnetic dipole moment that experiences a torque which allignes it with field. The tissue is illuminated with radio waves just the right frequency to flip this moments out of allignement.

The extent to which these radio-waves are absorbed in the tissue are proportional with the amount of H present.

$\Rightarrow$ chemical contrast, ideal for analysis soft tissues where X-Ray imaging does not work.

Magnetic dipoles and how magnets work

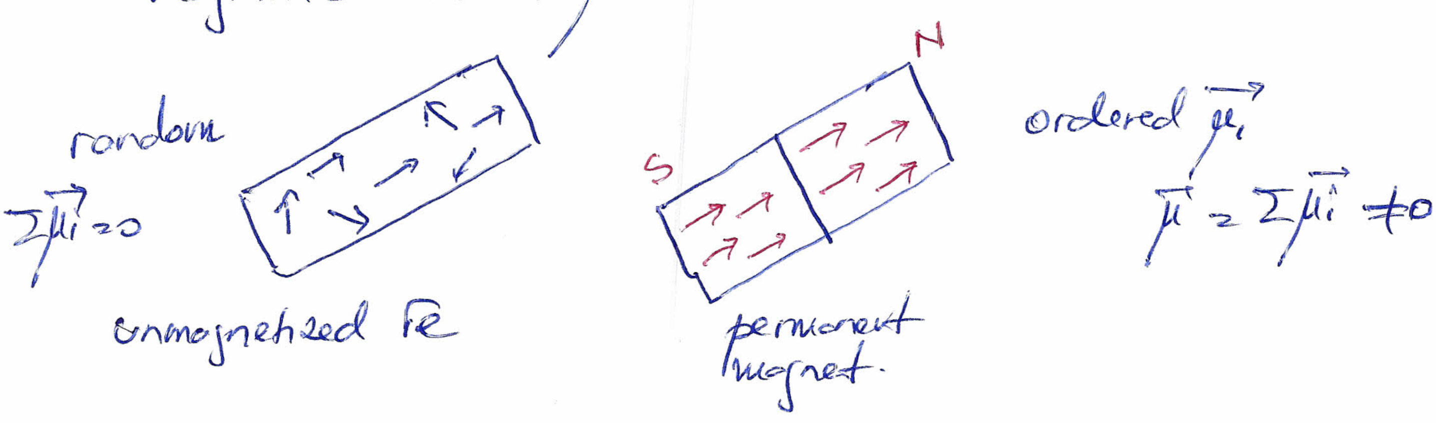
Think of an electron as being like a spinning ball of charge $\Rightarrow$ the circulation of charge around the spinning axis $\Leftrightarrow$ current loop $\Rightarrow$ e^- will have a net magnetic moment.

(explanation not QM exact but... intuitive here).

In some atoms (e.g. Fe, Co, Ni ...) a substantial fraction of electron's magnetic moments allign one-each-other $\Rightarrow$ the atom has a non-zero magnetic moment. (In contrast, in most materials the total magnetic moment is zero)

In an unmagnetized piece of Fe there is no overall alignement of atom's magnetic moment $\Rightarrow$ zero vector sum of moments

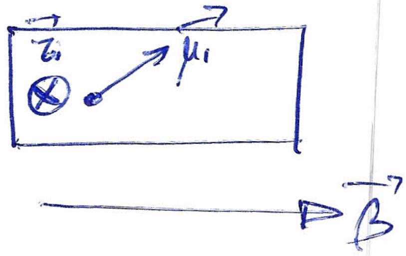
In a magnetized Fe bar, the magnetic moment of many atoms is parallel $\Rightarrow$ there is a substantial NET magnetic moment $\vec{\mu}$



If the magnet is placed in a magnetic field $\vec{B}$ that exerts a torque $\vec{\tau}$ on it $\Rightarrow$ allignement of magnet with $\vec{B}$ to minimize torque.

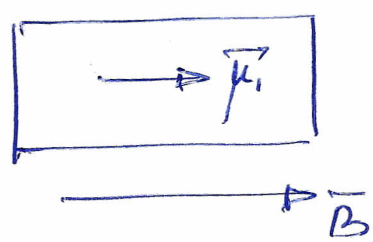
This torque explains the magnetization of unmagnetized objects:

$\sum \vec{\mu_i} = 0$ unmagnetized



$\vec{\tau_i} = \vec{\mu_i} \times \vec{B}$ which tends to allign $\vec{\mu_i} \parallel \vec{B}$ so that $U = -\vec{\mu_i} \cdot \vec{B} = 0$ (minimize potential energy)

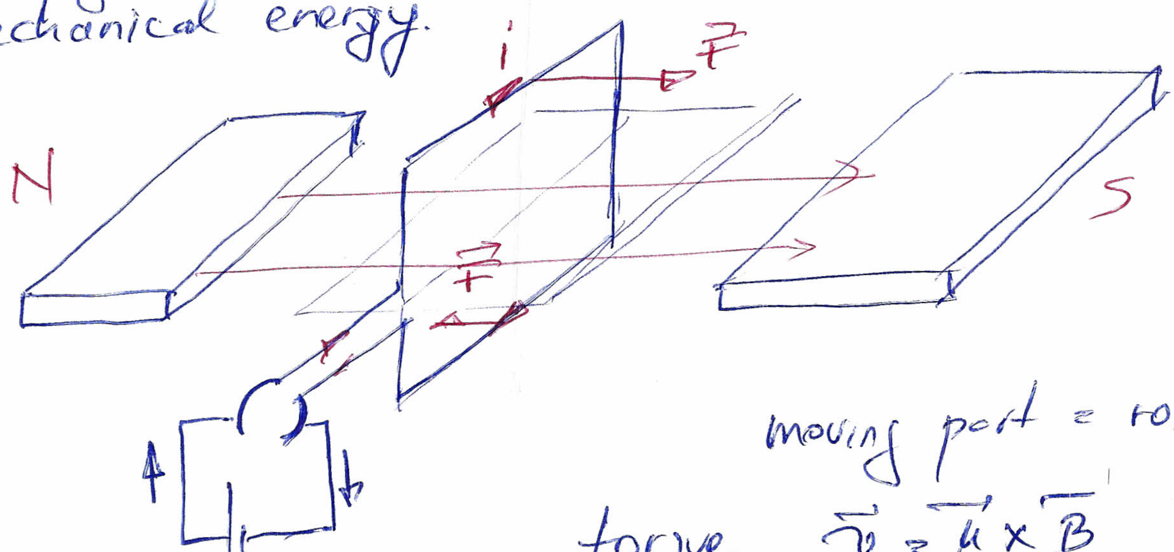
$\Rightarrow$ magnetized $\sum \vec{\mu_i} = \vec{\mu} \neq 0$



Obs: More details in the section concerning the magnetic properties of the matter.

(7) The Direct-Current Motor

In a motor, a magnetic torque acts on a current carrying conductor, and electric energy is converted into mechanical energy.

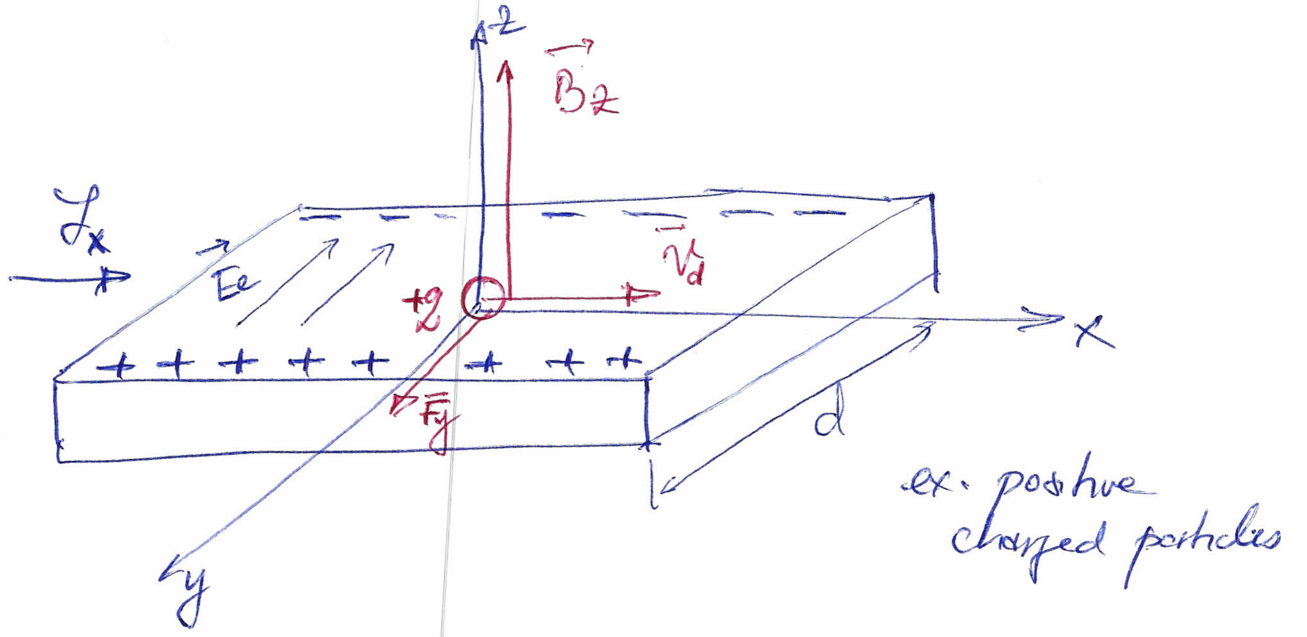


moving part = rotor

torque $\vec{\tau} = \vec{\mu} \times \vec{B}$

8 The Hall effect

→ discovered by Edwin Hall (1879) while he was still a graduate student



The magnetic force $\vec{F}$ acting on charged particles q moving with $\vec{v}_d$ in ⊥ $\vec{B}$ is:

$$F_y = |q| v_d B$$

An excess of positive charges accumulates at one edge and negative charge at the other edge. This accumulation continues until resulting transverse electric field compensates the magnetic force: ⇒

$$|q| E_e = |q| v_d B_z$$

This electric field causes a transverse potential difference ⇒ HALL VOLTAGE or HALL EMF

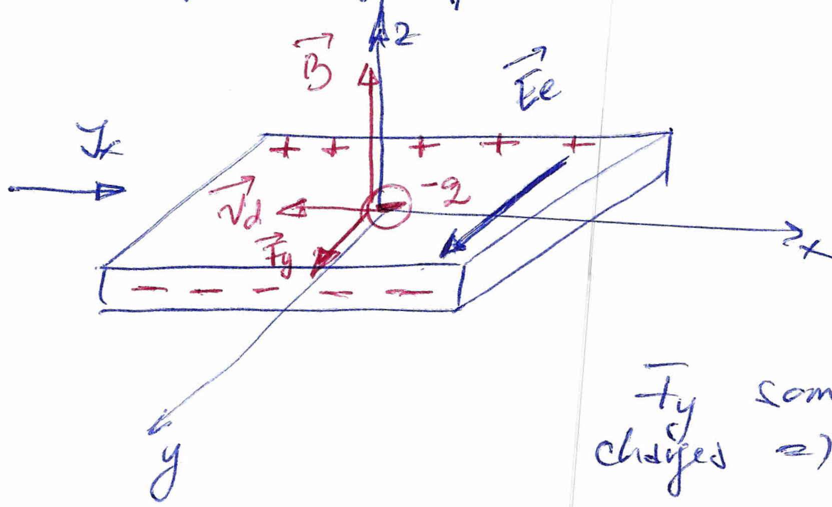
if $J_x = nq v_d$ ⇒ $\boxed{nq = \frac{-J_x B_z}{E_z}}$

and projected on oy: $-qE_e - q v_d B_z = 0$ (see fig).

Obs ① One can measure $E_y = \frac{U_{Hall}}{d}$,

and knowing j_x, B_z ⇒ the carriers concentration n can be determined

② The sign of charges is determined by the polarity of the Hall voltage



F_y same orientation as for + charges ⇒ e^- accumulates on edge (+y)

Obs ① In metals n is 3 order of magnitudes larger than in semiconductors.

- In metals the carriers are e^- ⇒ negative charges
- In SC carriers can be e^- ($q < 0$)
  holes ($q > 0$).

② In magnetic materials ⇒ anomalous Hall effect ⟶ ≈ 10 times larger V_{Hall} than in normal metals (intrinsic mechanisms ⟺ band structure and extrinsic mechanisms ⟺ scattering).